**Homework 2**

1. You are concentrating a solution containing a polypeptide by pressure filtration through a membrane (the polypeptide cannot pass through the membrane). The solute concentration on the feed side of the membrane is 0.1 M (1M = 1 mol/L); assume the temperature of the solution is 25oC. The applied pressure on the feed side of the membrane is 6 atm, and the pressure on the opposite side of the membrane is 1 atm. Assume the hydraulic conductance of the membrane is 3 mL hr-1 m-2 mmHg-1, and that the total surface area of the membrane is 1 m2. Determine **(A)** the net filtration pressure difference across the membrane, and **(B)** the filtration rate of the solvent (assume water).***(Ans: A = 2.54 atm = 1941 mmHg)***

**Ans:**

Here,

Hydraulic conductance,

Total surface area of membrane

Osmotic pressure

Osmotic pressure difference

We can relate flow and pressure using

1. The cell membrane is permeable to many different ions. Therefore, the equilibrium membrane potential for the case of multiple ions will depend not only on the concentrations of the ions within and outside the cell, but also on the permeability of the cell membrane to each ion (). The Goldmane equation shown, builds off of the Nernst equation to calculate the equilibrium membrane potential for the case of multiple ions. When considering Na+ and K+ ions, the equation is:

Where the subscripts *i* and *o* represent the inside and outside concentrations, respectively (hint: *z* is still equal to +1 even though two ions are considered). For a cell at rest, is much smaller than , with . During the depolarization stage of an action potential, the sodium permeability increases drastically, such that . Determine **(A)** the equilibrium membrane potential for a cell at rest, and **(B)** for a cell during the depolarization stage. Assume the concentrations of sodium and potassium in the cell are 14 mM and 140 mM, respectively. Assume the concentrations of sodium and potassium outside the cell are 140 mM and 4 mM, respectively. Assume the system is at physiological conditions. **(Ans: A = -85 mV)**

**Ans:**

At physiological conditions for a univalent ion (z=1)

At rest:

During depolarization:

1. Consider a horizontal hollow tube with a diameter of 20 mm filled with water. The water flowing through the tube (in the + z-direction) experiences a pressure drop of 2 mmHg over a length of 50 mm. Any effects from gravity are negligible. (A) Derive an expression for the axial velocity profile as a function of its radial position uz(r). (B) Determine the volumetric flow rate of water in the tube under these conditions (i.e., give an actual value)? (C) Assume the outer wall of the cylinder can move in the negative-z direction. Determine the velocity that would create a net flow rate of 0. **(Ans: C = 66.7 m/s)**

**Ans:**

Part A

The axial-direction (z-direction) Navier-Stokes equation for incompressible, steady, fully-developed, axisymmetric flow with negligible gravity is:

Assumptions:

* No radial and tangential velocities:
* Axisymmetric:
* Fully-developed flow:
* Steady-state:
* No gravity effects:

Simplified equation:

Integrate once:

Boundary conditions to determine constants:

**Axisymmetric condition** at : the velocity gradient at the center is zero.

Thus,

Divide by r, integrate again:

Integrate once more:

1. **No-slip boundary condition** at the outer wall

For general case:

Thus:

Hence, the general velocity profile is:

The wall is stationary (), so the velocity profile simplifies to the classic Hagen–Poiseuille profile:

Part B

Volumetric flow rate QQ is found by integrating velocity over the cross-sectional area:

Substitute velocity profile from Part A:

Evaluate integral explicitly:

Pressure gradient relation , we have Poiseuille's Law:

Given numerical values:

Calculating numerically provided:

Part C

Now, let the outer wall move in the negative z-direction at velocity . To achieve zero net flow:

Use the general velocity profile including :

Second integral evaluated explicitly:

Thus:

Solve for :

Numerically, with given values:

Thus,

1. Hemoglobin has a MW of 64,460. Estimate its molecular radius in nm, and estimate the diffusivity in water at 37 °C from both **(A)** the Renkin and Curry empirical equation (shown in class) and **(B)** the Stokes-Einstein equation. Assume ρ=1 g/cm3 and μ=0.76 cP. (Hint: for one answer you will need to find the diffusion coefficient first then the molecular radius; for the other answer you will find the molecular radius first, then the diffusion coefficient). Compare the radii values you calculated to values reported by online literature sources. **(Ans: a = 2.94 nm -> D = 10.16 x 10-7 cm2/s)**

Ans:

**Part A**

Renkin-Curry empirical relationship for diffusivity (D, in cm²/s):

Substitute the molecular weight of hemoglobin (MW = 64,460):

Performing the calculation provided:

The Stokes-Einstein equation to relate molecular radius aa and diffusion coefficient D:

Part B:

Assuming the protein (hemoglobin) is spherical with given density , the radius can be estimated from its volume and molecular weight:

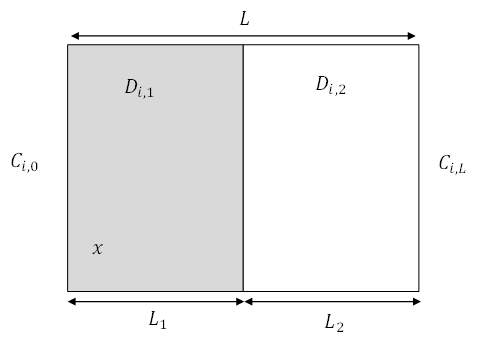
The formula to find the radius of a sphere given its molecular weight and density is:

Now, calculate D using Stokes-Einstein with the radius from above ():

**Comparison with Literature:**

* **Radius:** typically reports hemoglobin diameter ~5 nm
  + Renkin-Curry/Stokes-Einstein approach: 4.81 nm which is closer to literature.
  + Density-based Stokes-Einstein approach: 2.845 nm
* **Diffusion coefficient (D):** typical literature value ~6.4×10⁻⁷ cm²/s
  + Renkin-Curry: which is close to literature.
  + Stokes-Einstein (density-based):

1. Many biological tissues have layers with extracellular matrix components and different orientations of these components. As a result, diffusion coefficients vary from region to region. Consider the steady-state, one-dimensional diffusion of a protein across a tissue that consists of a cellular phase and an acellular phase (like an artery wall consisting of a layer of smooth muscle cells and a layer of elastic lamina. Assume no reactions occur in either layer. The protein diffusion coefficients in the layers (1 and 2) are *Di,1*and *Di,2*. The concentration at one edge is (x = 0) Ci = C0; on the other edge (x = L1 + L2 = L) Ci = CL. Assume that all partition coefficients in both layers are equal to 1. Use the figure below to help. Determine **(A)** the concentration as a function of position *x*, **(B)** the flux of the protein solute across the tissue, and **(C)** the effective diffusion coefficient if the system is modeled as a single layer. **(Hint: at the interface (x = L1), the concentrations will be equal and the flux will be equal.)**



**Ans:**

Part A

Step 1: Fick's Second Law

The problem is steady-state, meaning concentration does not change with time. Hence:

From Fick’s second law:

Since diffusion coefficient is nonzero and constant within each layer, the above simplifies to:

This implies a linear concentration profile in each layer.

Thus, for two layers, the concentration profiles have the forms:

* Layer 2:

Step 2: Apply Boundary Conditions

We have four unknown constants . We use four boundary conditions:

1. Concentration at the left boundary:

1. Concentration at the right boundary:
2. Continuity of concentration at the interface x=L1x = L\_1:
3. Continuity of flux at the interface :

Thus:

Step 3: Solve for the Constants

From condition 3, substitute and the expression for :

Solve this equation for :

Thus:

Step 4: Substitute constants to get concentration profiles

Thus, the final concentration profiles are:

Layer 1 ():

Layer 2 ():

Part B: Determining Flux Across the Layers

The flux must be equal through both layers at steady state. Using Fick’s law (), calculate the flux:

In layer 1:

In layer 2:

Because the system is at steady state, fluxes must match :

The unified flux expression is:

Part C: Effective Diffusion Coefficient

If we consider both layers as a single equivalent layer (length L) with an effective diffusion coefficient :

Set equal to the previously calculated flux: